Math 115

Winter 2017

Lecture 17

① Solve by Quadratic formula:

$$5x^2 + 8x = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$5x^2 + 8x + 3 = 0$$

$$x = \frac{-8 \pm \sqrt{4}}{2(5)}$$

$$b^2 - 4ac = 8^2 - 4(5)(3)$$

$$= 64 - 60$$

$$= 4$$

$$= \frac{-8 \pm 2}{10}$$

$$x = \frac{-8 + 2}{10}$$

$$x = \frac{-6}{10} = \boxed{\frac{-3}{5}}$$

$$x = \frac{-8 - 2}{10}$$

$$= \frac{-10}{10}$$

$$= \boxed{-1}$$

$$\left\{ \frac{-3}{5}, -1 \right\}$$

② Reduce: $\dfrac{x^2 - 25}{x^2 - 2x - 15}$

$$= \frac{(x+5)(x-5)}{(x-5)(x+3)} = \frac{x+5}{x+3}$$

① find all **excluded** values, then Simplify:

$$\frac{x^2 + 2x}{2x^2 + x - 6}$$

$\hookrightarrow$ Deno. $= 0$, Solve

$$2x^2 + x - 6 = 0$$

$a = 2 \quad b = 1 \quad c = -6$

$b^2 - 4ac = 1^2 - 4(2)(-6)$

$= 49$

$$= \frac{x(\cancel{x + 2})}{(2x - 3)(\cancel{x + 2})}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \boxed{\frac{x}{2x - 3}}$$

$$= \frac{-1 \pm \sqrt{49}}{2(2)} = \frac{-1 \pm 7}{4}$$

E.V.
$\frac{3}{2}, -2$

$x = \frac{-1 + 7}{4} = \frac{6}{4} = \boxed{\frac{3}{2}}, \quad x = \frac{-1-7}{4} = \boxed{-2}$

Divide:

$$\frac{2x^2 + 59x - 30}{x^2 - 10x + 25} \div \frac{2x^2 - 21x + 10}{x^2 + x - 30}$$

$$= \frac{2x^2 + 59x - 30}{x^2 - 10x + 25} \cdot \frac{x^2 + x - 30}{2x^2 - 21x + 10}$$

$$= \frac{(2x - 1)(x + 30)}{(x - 5)(x - 5)} \cdot \frac{(x - 5)(x + 6)}{(2x - 1)(x - 10)}$$

$$= \boxed{\frac{(x + 30)(x + 6)}{(x - 5)(x - 10)}}$$

Simplify:

① $\dfrac{x-5}{x^2-9} + \dfrac{2}{x^2-9}$

$= \dfrac{x-5+2}{x^2-9}$

$= \dfrac{x-3}{x^2-9} = \dfrac{1(x-3)}{(x+3)(x-3)}$

$= \boxed{\dfrac{1}{x+3}}$

② $\dfrac{3-2x}{x^2-6x+8} - \dfrac{3x-7}{x^2-6x+8}$

$= \dfrac{3-2x-3x+7}{x^2-6x+8}$

$= \dfrac{-5x+10}{x^2-6x+8}$

$= \dfrac{-5(x-2)}{(x-2)(x-4)}$

$= \boxed{\dfrac{-5}{x-4}}$

Simplify:

$\dfrac{x-1}{4x} - \dfrac{4(2x+3)}{4x} = \dfrac{x-1-8x-12}{4x} = \boxed{\dfrac{-7x-13}{4x}}$

$\dfrac{x}{x+3} - \dfrac{1}{x-1} = \dfrac{x(x-1)}{(x+3)(x-1)} - \dfrac{1(x+3)}{(x-1)(x+3)}$

$= \dfrac{x^2-x-x-3}{(x+3)(x-1)}$

$= \boxed{\dfrac{x^2-2x-3}{(x+3)(x-1)}} = \boxed{\dfrac{(x+1)(x-3)}{(x+3)(x-1)}}$

Simplify

Hint:
Factor Denominators
Find LCD

$$\frac{6}{x^2+x-2} + \frac{4}{x^2-4x+3}$$

$$= \frac{6(x-3)}{(x+2)(x-1)(x-3)} + \frac{4(x+2)}{(x-1)(x-3)(x+2)}$$

$$= \frac{6x-18+4x+8}{(x+2)(x-1)(x-3)} = \frac{\overset{10(x-1)}{10x-10}}{(x+2)(x-1)(x-3)}$$

$$= \frac{10}{(x+2)(x-3)}$$

Simplify

$$\frac{x}{x^2+17x+72} - \frac{1}{x^2+15x+56}$$

$$= \frac{x(x+7)}{(x+8)(x+9)(x+7)} - \frac{1(x+9)}{(x+7)(x+8)(x+9)}$$

$$= \frac{x^2+7x-1x-9}{(x+8)(x+9)(x+7)} = \boxed{\frac{x^2+6x-9}{(x+8)(x+9)(x+7)}}$$

Use LCD to clear fractions, then Solve

$\dfrac{x}{8} - \dfrac{8}{x} = 0$ LCD = 8x

$8x \cdot \dfrac{x}{8} - 8x \cdot \dfrac{8}{x} = 8x \cdot 0$

$x^2 - 64 = 0 \implies (x - 8)(x + 8) = 0$

by Z.P.R.

$x - 8 = 0$ or $x + 8 = 0$

$\boxed{x = 8}$ $\boxed{x = -8}$

$\{\pm 8\}$

Solve;

$x - \dfrac{35}{x} = 2$, LCD = x

$x^2 - 35 = 2x$

$x^2 - 2x - 35 = 0$

$(x - 7)(x + 5) = 0$

by Z.F.P. $\implies x - 7 = 0 \implies x = 7$ $\{7, -5\}$

$x + 5 = 0$ $x = -5$

Solve

$$\boxed{\frac{1}{x-3}} + \boxed{\frac{1}{x+3}} = \boxed{\frac{6}{x^2-9}}$$

LCD = $(x-3)(x+3)$

$x + \cancel{3} + x - \cancel{3} = 6$

$2x = 6 \rightarrow \cancel{x=3}$

→ however, 3 is an E.V.

$\emptyset$
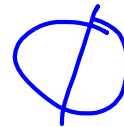
Solve

$$\boxed{\frac{3}{x-5}} - \boxed{\frac{2}{x+3}} = \boxed{\frac{15}{x^2-2x-15}}$$

LCD = $(x-5)(x+3)$; E.V. 5 & -3

$3(x+3) - 2(x-5) = 15$

$3x+9 -2x+10 = 15$

$x + 19 = 15$

$\boxed{x=-4}$ $\{-4\}$

Solve

$$\boxed{\frac{4}{x-6}} - \boxed{\frac{3}{x+8}} = \boxed{\frac{56}{x^2+2x-48}}$$

$LCD = (x-6)(x+8)$; E.V. $6 \overset{\vdots}{=} -8$

$4(x+8) - 3(x-6) = 56$

$4x + 32 - 3x + 18 = 56$

$x + 50 = 56 \rightarrow \boxed{x=6}$ ⊘

Bart can clean the garage in 2 hrs alone, while Homer can do the same job in 3hrs alone. How long if they work together?

Work by Bart	+	Work by Homer	=	1 Complete Work	→ Solve

$\frac{1}{2} \cdot t + \frac{1}{3} \cdot t = 1$

$\frac{t}{2} + \frac{t}{3} = 1$

$LCD = 6$

$3t + 2t = 6$

$5t = 6$

$t = \frac{6}{5}$

$t = 1.2$ hrs

1.2 hrs

It takes Batman 5 minutes longer than Superman to wash the dishes.

Together, they can do it in 6 minutes.

How long if they work alone?

Superman $\rightarrow x$ minutes $\rightarrow$ his rate is $\frac{1}{x}$

Batman $\rightarrow x+5$ minutes $\rightarrow$ his rate is $\frac{1}{x+5}$

$$\begin{array}{ccccc} \text{work} & & \text{work} & & 1 \\ \text{by} & + & \text{by} & = & \text{Complete} \\ \text{Superman} & & \text{Batman} & & \text{work} \end{array}$$

$$\frac{1}{x} \cdot 6 + \frac{1}{x+5} \cdot 6 = 1 \Rightarrow \text{Solve} \quad \frac{6}{x} + \frac{6}{x+5} = 1$$

$\Rightarrow$ Solve $\boxed{\frac{6}{x}} + \boxed{\frac{6}{x+5}} = \boxed{1}$

$\rightarrow x^2 + 5x - 12x - 30 = 0$

$x^2 - 7x - 30 = 0$

$(x - 10)(x + 3) = 0$

LCD $= x(x+5)$

$6(x+5) + 6x = x(x+5)$

$6x + 30 + 6x = x^2 + 5x$

$12x + 30 = x^2 + 5x$

$\boxed{x = 10}$ $\cancel{x = -3}$

Superman in 10 minutes
&
Batman in 15 minutes

It takes Maria 3 hrs shorter than it takes Mike to do a certain job.

It takes both of them 2 hrs to do the job if they work together.

How long if they work alone?

Time:
Mike → x hrs
Maria → x−3 hrs

Rate:
Mike $\frac{1}{x}$
Maria $\frac{1}{x-3}$

work by Maria + work by Mike = one Complete work

$\frac{1}{x-3} \cdot 2 + \frac{1}{x} \cdot 2 = 1$

Solve

$\frac{2}{x-3} + \frac{2}{x} = 1$

Solve

$\frac{2}{x-3} + \frac{2}{x} = 1$

LCD = (x−3)·x

$2x + 2(x-3) = (x-3) \cdot x$

$2x + 2x - 6 = x^2 - 3x$

$x^2 - 3x - 4x + 6 = 0$

$x^2 - 7x + 6 = 0$

$(x-1)(x-6) = 0$

~~x=1~~ $x=6$

Mike in 6 hrs
&
Maria in 3 hrs

Mr. Spock & Captain Kirk can defeat the enemy in 21 days if they work together.

It takes Captain Kirk ↑takes 3 times longer than Mr. Spock to defeat same enemy fighting alone.

Mr. Spock in 28 Days
Cap. Kirk in 84 Days

How long if they work alone?

Rate:

Kirk $\frac{1}{3x}$

Spock $\frac{1}{x}$ $\Rightarrow$

$\frac{1}{3x} \cdot 21 + \frac{1}{x} \cdot 21 = 1$

$\frac{21}{3x} + \frac{21}{x} = 1$

Mr. Spock	Cap. Kirk
1	3·1
5	3·5
x	3x

$\frac{7}{x} + \frac{21}{x} = 1$

$\boxed{x = 28}$

It takes 10 minutes to fill up the sink, and 12 minutes to empty the sink.

If water is running, and drain left open, How long to (fill up) the sink?

$\frac{1}{10} \cdot t - \frac{1}{12} \cdot t = 1$

Solve $\frac{t}{10} - \frac{t}{12} = 1$

LCD = 60

$6t - 5t = 60$

$\boxed{t = 60}$

(60 minutes)

① Q-formula

② Excluded Values

③ +, −, •, ÷ with rational expressions

④ Solve Rational Equations

⑤ Work Problems

⑥ Review exams 1, 2, and 3.

Simplify

$$\frac{\frac{2}{3} - \frac{1}{2}}{\frac{3}{4}} = \frac{12 \cdot \frac{2}{3} - 12 \cdot \frac{1}{2}}{12 \cdot \frac{3}{4}} = \frac{8 - 6}{9} = \boxed{\frac{2}{9}}$$

LCD = 12

$$\frac{x - \frac{4}{x}}{1 + \frac{2}{x}} = \frac{x \cdot x - x \cdot \frac{4}{x}}{x \cdot 1 + x \cdot \frac{2}{x}} = \frac{x^2 - 4}{x + 2}$$

LCD = x

$$= \frac{(x - 2)(x + 2)}{x + 2} = \boxed{x - 2}$$

Simplify

$$\frac{1 + \frac{7}{x} + \frac{12}{x^2}}{1 - \frac{16}{x^2}} = \frac{x^2 + 7x + 12}{x^2 - 16}$$

LCD $= x^2$

$$= \frac{(x+3)(x+4)}{(x+4)(x-4)} = \frac{x+3}{x-4}$$

Simplify:

$$\frac{x - 2 + \frac{1}{x}}{x - 5 + \frac{4}{x}} = \frac{x^2 - 2x + 1}{x^2 - 5x + 4} = \frac{(x-1)(x-1)}{(x-4)(x-1)}$$

$$= \boxed{\frac{x-1}{x-4}}$$

Complex
Rational
Expressions

Simplify

$$\boxed{\frac{5}{x-3}} - \boxed{\frac{2}{x+7}} = \frac{5(x+7) - 2(x-3)}{3x+41}$$

$$\boxed{\frac{3x+41}{x^2+4x-21}}$$

LCD = $(x-3)(x+7)$

we must be

aware of values that make this
  problem undefined or indeterminate.

$$= \frac{5x+35-2x+6}{3x+41}$$

$$= \frac{3x+41}{3x+41} = \boxed{1}$$

Simplify:

$$\frac{\dfrac{x}{x-2} - \dfrac{1}{x+3}}{\dfrac{1}{x^2+x-6}} = \frac{x(x+3)-1(x-2)}{1}$$

LCD = $(x-2)(x+3)$

Review
Recent
Notes

$$= \frac{x^2+3x-x+2}{1}$$

$$= \boxed{x^2+2x+2}$$

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Final exam:

6:00 – 10:30

You can come in as late as 7:00.

Math 125 ASAP.